Preface

The aim of this book is to provide an introduction to studying the solvability of infinite systems of semilinear differential-functional parabolic equations in partially ordered Banach spaces by using monotone iterative methods and topological methods. The infinite systems of ordinary differential equations, integro-differential equations and differential-functional equations are natural generalizations of finite systems of these equations and we note that these systems play a special role in the mathematical modelling of numerous difficult real world problems.

The book is divided into six chapters. Chapter 1 is devoted to nonlinear systems of parabolic inequalities in which Volterra functionals arise. The theorems on weak and strong differential-functional inequalities, the comparison theorem and a uniqueness criterion are given.

In Chapters 2 and 4, monotone iterative methods are applied to studying the existence and uniqueness of the classical solutions of the Fourier first initial-boundary value problem for the infinite systems of semilinear differential-functional parabolic equations. However, while applying the method of upper and lower solutions, we assume the monotonicity of the right-hand sides of the systems with respect to unknown functions, and the existence of an ordered pair of a lower and an upper solution for the problem considered. These assumptions are not typical of existence and uniqueness theorems, but monotone iterative methods are constructive in nature and we obtain two sequences of successive approximations which tend monotonically — one from above and the other from below — to the sought-for exact solution and we obtain also *a priori* information on their localization. In Chapter 3 we present the important truncation method for infinite countable systems.

This problem is also studied with topological methods in Chapter 5. To prove the existence of the solutions, the Banach, Schauder and Leray-Schauder fixed point theorems are used. We remark that the *a priori* estimates which appear while applying these fixed point theorems are correspond, respectively, to the above mentioned assumptions in the theory of monotone iterative meth-

ods. In Chapter 6, the existence of a solution of the Dirichlet problem for an infinite system of elliptic equations is studied. The results obtained are applied to studying the asymptotic stability of solutions of parabolic systems.

In Appendix, we give some auxiliary theorems, including fundamental theorems on the existence and uniqueness of solutions of linear parabolic and elliptic problems and for their integral representations, as well as classical theorems on the Hölder and Sobolev spaces of natural order and on relations between them, expressed as imbedding theorems.

This monograph is based on the results of my 20 papers. The presented results generalize my earlier results and other results given by several authors in numerous papers for finite systems of parabolic differential equations, to encompass the case of infinite systems of parabolic differential-functional equations. The bibliography contains about 200 entries.

My interest in problems of parabolic differential equations, differential inequalities, monotone iterative methods and its applications to the mathematical modelling of real world problems has been inspired by Professors Włodzimierz Mlak, Mirosław Krzyżański, Jacek Szarski and Tadeusz Ważewski, with the preeminent role of Professor Jacek Szarski.

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